Sun Navigation

1 To compute a location at sea with the formulas of C. F. Gauss

1.1 Inputs Circle of Position 1 (COP 1)

1. Date
2. Time of observation
3. Sextant reading

From these three inputs, the database for the Sun Almanac provides:
- GHA Greenwich angle, generated from date and first observation time and.
- δ Declination, generated from date and first observation time.

The databases provide values for the correction of the sextant reading:
- h Observed altitude of the sun in first observation

1.2 Calculations Step one

With these three values A, B and C the following calculation starts without branching:

(1) $\theta = K \cdot (GHA' - GHA)$ (K: look 1.4 point 4.)

(2) $F = \tan^{-1} \dfrac{\tan \delta'}{\cos(\theta)}$

(3) $V = \tan^{-1} \dfrac{\cos F \cdot \tan \theta}{\sin(F - \delta)}$

(4) $W = P \cdot \cos^{-1} \dfrac{\cos V \cdot \tan h}{\tan(F - \delta)} \left(\dfrac{\sin h' \cdot \sin F}{\sin h \cdot \sin \delta' \cdot \cos(F - \delta)} - 1 \right)$

(5) $G = \tan^{-1} \dfrac{\tan h}{\cos(V - W)}$

(6) $\varphi = \tan^{-1}(\cos \tau \cdot \cot(G - \delta)$

(7) $\tau = \tan^{-1} \dfrac{\cos G \cdot \tan(V - W)}{\sin(G - \delta)}$

(8) $\lambda^* = Grt + K \cdot \tau$

- Simply calculate equations 1 to 8 one after the other. In the following only φ and λ^* are needed.
- If *Approximate latitude* (Settings) > δ then P = 1 / if *Approximate latitude* < δ then P = -1.
- K = 1. A second observation in an earlier time of day on a following day is possible with K = -1.
- F, V, W, G are substitutions within this formula package.

Graphic: green solid COP $\lambda(\varphi) = GHA \pm \cos^{-1} \dfrac{\sin h - \sin \delta \cdot \sin \varphi_x}{\cos \delta \cdot \cos \varphi_x}$; $\varphi_x \in [-\pi ; \pi]$

1.3 Inputs Circle of Position 2 (COP 2)

1. Date
2. Time of observation
3. Sextant reading

From these three inputs, the database for the Sun Almanac provides:

- GHA' Greenwich angle, generated from date and second observation time and.
- δ' Declination, generated from date and second observation time.

The databases provide values for the correction of the sextant reading:

- h' Observed altitude of the sun in second observation

1.3.1 Including a change of location between the observations:

(9) $$LHA = Grt - \lambda^*; \quad 0 \leq LHA \leq 2\pi$$ (LHA = Local Hour Angle)

(10a) $$z = \arccos \frac{\sin \delta - \sin \varphi \cdot \sin h}{\cos \varphi \cdot \cos h} \quad \text{if } LHA > \pi$$

(10b) $$z = 2\pi - \arccos \frac{\sin \delta - \sin \varphi \cdot \sin h}{\cos \varphi \cdot \cos h} \quad \text{if } LHA < \pi$$

(11) $$hs = h + \frac{d}{60} \cdot \cos(z - c)$$

d = DMG in nautical miles (NM) and c = CMG from Dead Reckoning. In the case of *Specify* see Eq. 22.

Graphic: green dashed COP $\lambda(\varphi) = GHA \pm \cos^{-1} \frac{\sin h_s - \sin \delta \cdot \sin \varphi_x}{\cos \delta \cdot \cos \varphi_x}; \quad \varphi_x \in [-\pi; \pi]$

1.3.2 The final position is found by calculating equations 4 to 8 again, but this time using hs:

(12) $$Ws = P \cdot \cos^{-1} \frac{\cos V \cdot \tan hs}{\tan(F - \delta)} \left(\frac{\sin h' \cdot \sin F}{\sin hs \cdot \sin \delta' \cdot \cos(F - \delta)} - 1 \right)$$

(13) $$Gs = \tan^{-1} \frac{\tan hs}{\cos(V - W)}$$

(14) $$\varphi s = \tan^{-1}(\cos \tau s \cdot \cot(Gs - \delta)$$ (Latitude of the position)

(15) $$\tau s = K \cdot \tan^{-1} \frac{\cos G \cdot \tan(V - Ws)}{\sin(Gs - \delta)}$$ (K: look 1.4 point 4.)

(16) $$\lambda^* s = Grt + \tau s$$

(17) $$\lambda s = \begin{cases} -\lambda^* s & \text{wenn} \quad 0 < \lambda^* s < \pi \\ 2\pi - \lambda^* s & \text{wenn} \quad \pi < \lambda^* s < 2\pi \end{cases}$$ (Longitude of the position)

- K = 1. A second observation in an earlier time of day on a following day is possible with K = -1.
- Replace approximate altitude by φs.

Graphic: red solid COP $\lambda(\varphi) = GHA' \pm \cos^{-1} \frac{\sin h' - \sin \delta' \cdot \sin \varphi_x}{\cos \delta' \cdot \cos \varphi_x}$; $\varphi_x \in [-\pi; \pi]$

1.4 Observations on the same and following days:

- Date of observation 1: d
- Date of observation 2: d'
- Time of observation 1: ot
- Time of observation 2: ot'
- Intermediate time: $\Delta t = d' + ot' - (d + ot)$

$\Delta t_r = d' + ot' - (d + ot) - \mathbb{Z}\big(d' + ot' - (d + ot)\big)$ is ever less than 1, because $\mathbb{Z}(x)$ means integer of x.

How to deal with it:

1. if $\Delta t_r < 0$ → Error, do not perform calculations
2. if $\Delta t_r < 0$ → Error, do not perform calculations
3. if $0 \leq \Delta t_r \leq$ 6:00 h → okay, nothing more required
4. if $\Delta t_r >$ 6:00 h → multiply eq. 1 and eq. 15 with -1
5. if d' - d > 3 → Error, do not perform calculations

2 Position calculation with the noon latitude

2.2 Inputs for Noon Altitude

1. Date
2. Time of observation
3. Sextant reading
4. Sun Direction N / S

From these four inputs, the database for the Sun Almanac provides:

- GHA or GHA' Greenwich angle, generated from date and observation times.
- δ or δ' Declination, generated from date and observation times.

The databases provide values for the correction of the sextant reading:

- h, h' or h_N Observed altitude of the sun in first and second observation.

2.3 In case of 1st. observation „Circle of Position 1“ and second observation „Noon latitude“

Determine the altitude of the sun in a sufficient time before noon of the ship and enter the data. That's all for now.

Graphic: green solid COP: $\lambda(\varphi) = GHA \pm \cos^{-1} \frac{\sin h - \sin \delta \cdot \sin \varphi_x}{\cos \delta \cdot \cos \varphi_x}$; $\varphi_x \in [-\pi; \pi]$

Some time later at ship's midday calculate the noon latitude φ_N from the noon altitude h_N of the sun:

(18)
$$\varphi_N = \begin{cases} (\pi/2 - h_N) + \delta & \text{sun bearing S} \\ (h_N - \pi/2) + \delta & \text{sun bearing N} \end{cases}$$

Graphic: The color of the noon latitude is yellow.

And with that the calculation of the position longitude:

(19)
$$\lambda^* s = GHA + \cos^{-1} \frac{\sin hs - \sin\delta \cdot \sin\varphi_N}{\cos\delta \cdot \cos\varphi_N}$$

Eliminate carryovers by adding 2π if $\lambda^* < 0$ or subtracting 2π if $\lambda^* < 2\pi$.

With $\lambda^* < \pi$ it is west lengths, if $\lambda^* > \pi$, then λ^* is to be subtracted from 2π and we will get east lengths:

$$\lambda s = \begin{cases} -\lambda^* s & \text{wenn} \quad 0 < \lambda^* s < \pi \\ 2\pi - \lambda^* s & \text{wenn} \quad \pi < \lambda^* s < 2\pi \end{cases}$$

The position of the ship is φ_N / λs. Replace approximate altitude by φ_N.

2.4 In case of 1st. observation „Noon latitude“and second observation „Circle of Position 2“

Observing the culmination of the sun, entering the data and calculating the noon latitude. That's all for now.

(20)
$$\varphi_N = \begin{cases} (\pi/2 - h_N) + \delta' & \text{sun bearing S} \\ (h_N - \pi/2) + \delta' & \text{sun bearing N} \end{cases}$$

Graphic: The color of the noon latitude is yellow.

Some time later after second observation calculate the circle of Position 2:

(21)
$$\lambda^* s = GHA' - \cos^{-1} \frac{sin\ h' - \sin\delta' \cdot \sin(\varphi_N + \Delta\varphi)}{\cos\delta' \cdot \cos(\varphi_N + \Delta\varphi)}$$

In this Equation $\Delta\varphi$ comes from the Dead Reckoning module and is there the current value of $\Delta\varphi_T$.

In the case of *Specify* in the Dead Reckoning module, $\Delta\varphi$ is calculated as follows:

(22)
$$\Delta\varphi = d \cdot \cos c$$
(d = DMG, c = CMG, both in radian)

With $\lambda^* < \pi$ it is west lengths, if $\lambda^* > \pi$, then λ^* is to be subtracted from 2π and we will get east lengths:

$$\lambda s = \begin{cases} -\lambda^* s & \text{wenn} \quad 0 < \lambda^* s < \pi \\ 2\pi - \lambda^* s & \text{wenn} \quad \pi < \lambda^* s < 2\pi \end{cases}$$

The position of the ship is φ_N / λs. Replace approximate altitude by φ_N.

Graphic: red solid COP $\lambda(\varphi) = GHA' \pm \cos^{-1} \frac{\sin h' - \sin\delta' \cdot \sin\varphi_x}{\cos\delta' \cdot \cos\varphi_x}$; $\varphi_x \in [-\pi; \pi]$